Hyperbolic Geometry and Exploration of Mathematical Topology Through Crochet

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Abstract

In 1963, the hyperbolic plane was of extreme influence in the work of Dutch artist, M.C. Escher. Although Escher did not have any technical training in mathematics, he was still able to illustrate and appreciate mathematical concepts, creating a bridge between math and art.

By exploring hyperbolic geometry and mathematical topology through crochet, one can also attempt to create a bridge. One can use different stitch ratios used to create a hyperbolic plane to exemplify the effects of exponential growth. Crocheted surfaces of the hyperbolic plane also allows one to manipulate and interact with the surface to fully illustrate what a non-Euclidean surface entails.

Furthering one's exploration into topology, having two crocheted mobius bands that are mirror-imaged, one can not only illustrate the concept of non-orientable surfaces, but can demonstrate how a Klein bottle is formed, touching on 4-D topology.

References

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Introduction

In "Elements" written by Euclid, the "father of geometry" Euclid talks of 5 postulates which define a Euclidean surface, or a standard three-dimensional surface of the x, y, z plane. The five postulates of Euclid are:

1. A straight line can be drawn joining any two points
2. Any line segment can be extended indefinitely in a straight line
3. Given any straight line segment, a circle can be drawn having the segments as the radius and one of the endpoints as the center.
4. All right angles are congruent
5. Given a line and a point not on that line, there is exactly one line through that point exactly parallel to the original line, where the line can extend forever and never intersect with the original line

Where hyperbolic geometry differs from Euclidean geometry is in the 5th postulate. If parallel lines are drawn on a hyperbolic surface, the lines will eventually diverge in two directions. In fact, multiple lines can be drawn through the same point and never intersect with another line. This can be illustrated via the lines sewn through Figure A.

Mobius Strips can also be illustrated through crochet as seen in Figure B. Mobius Strips highlight a non-orientable surface, a surface which can be walked around in a "clockwise" fashion which will eventually change to walking in a "counterclockwise" fashion, and then allows the walker to return to the starting point.

By combining two mirror-image Mobius Strips, one can also create a Klein Bottle as illustrated in Figure C. When the two non-orientable surfaces are merged, they create a figure which intersects itself, creating a lack of distinction of where the figure stops and begins. This lack of boundary combined with its non-orientable surface allows a Klein bottle to be considered fourth-dimensional.

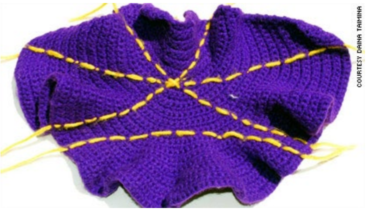

Figure A

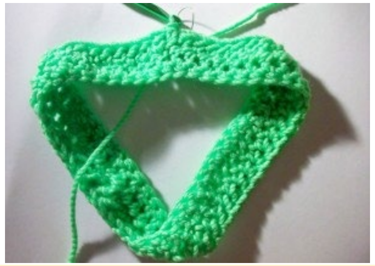

Figure B

Figure C

Construction Instructions

Hyperbolic Plane Construction:

1. Begin with 20 chain stitches
2. For each first stitch in each row, insert the hook into the second chain from the hook. Single Crochet.
3. For the next N stitches, where N represents the first number in the ratio that will occur for the rest of the crocheted hyperbolic plane, single crochet.
4. For the (N+1) stitch, increase.
5. Repeat Steps 3 and 4 until the end of the row.
6. At the end of the row, before going to the next row, do one extra chain stitch
7. Proceed to next row, remembering to begin at the second chain from the hook for each row.
8. When model is as big as desired, yarn through the last loop.

Mobius Strip Construction:

1. Begin with 50 chain stitches and twist one end 180 degrees
2. Join two ends, making a slip stitch in the bottom loop of the last chain.
3. Chain 3 and double crochet into the bottom loop of your chain stitches. Mark the beginning of the round. Continue to double crochet into the rest of the of the bottom loops of the chain until the marker is reached.
4. Double Crochet into the top loops of the initial chain stitches until the marker is reached. Single stitch into the chain-3.
5. Crochet around your mobius until the desired width is reached, then yarn through the last loop.